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# Hyperfine Splittings in SU(3) Skyrmion through BFT SU(2) Embedding Scheme

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## ABSTRACT

We apply the Batalin, Fradkin and Tyutin (BFT) formalism to the SU(3) flavor Skyrmion model to investigate the Weyl ordering correction to the structure of the hyperfine splittings of strange baryons. Differently from the Klebanov and Westerberg's standard rigid rotator approach to the SU(3) Skyrmion where the angular velocity of the SU(2) rotation was used, we have exploited the SU(2) collective coordinates which are naturally embedded in the SU(3) group manifold so that, as in the SU(2) flavor case, we can introduce the BFT scheme in the SU(3) Skyrmion to yield the modified baryon energy spectrum. The Berry phases and Casimir effects are also discussed.

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It is well known that baryons can be obtained from topological solutions, known as SU(2) Skyrmions, since the homotopy group $\Pi_3(SU(2)) = Z$ admits fermions [1, 2, 3]. Using the collective coordinates of the isospin rotation of the Skyrmion, Adkins et al. [1] have performed semiclassical quantization having the static properties of baryons within 30% of the corresponding experimental data. After their work, several authors have been tried to generalize the Skyrmion model to include the strange flavor degrees of freedom[4, 5]. On the other hand, recently it has been shown that a method [6] developed by Batalin, Fradkin and Tyutin (BFT), which converts the second class constraints into first class ones by introducing auxiliary fields, gives an additional energy term in SU(2) Skyrmion model[7].

The motivation of this paper is to extend the BFT scheme for the SU(2) Skyrmion to the SU(3) flavor case so that one can investigate the Weyl ordering correction to $c$ the ratio of the strange-light to light-light interaction strengths and $\bar{c}$ that of the strange-strange to light-light. Differently from the Klebanov and Westerberg's standard rigid rotator approach[4] to the SU(3) Skyrmion where the angular velocity of the SU(2) rotation was used, we exploit the SU(2) collective coordinates which are naturally embedded in the SU(3) group manifold so that, as in the SU(2) flavor case, one can introduce the BFT scheme in the SU(3) Skyrmion to yield the modified baryon energy spectrum and the structure of the hyperfine splittings of strange baryons.

Now we start with the SU(3) Skyrmion Lagrangian of the form

$$\begin{aligned} L &= \int \mathrm{d}^3r [\frac{f_\pi^2}{4}\mathrm{tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{32e^2}\mathrm{tr}[U^\dagger\partial_\mu U, U^\dagger\partial_\nu U]^2 \\ &\quad + \frac{f_\pi^2}{4}\mathrm{tr}M(U + U^\dagger - 2)] + L_{WZW}, \end{aligned} \tag{1}$$

where $f_\pi$ is the pion decay constant and $e$ is a dimensionless parameter, and $U$ is an SU(3) matrix. $M$ is proportional to the quark mass matrix given by

$$M = \mathrm{diag}\ (m_\pi^2,\ m_\pi^2,\ 2m_K^2 - m_\pi^2)$$

where $m_\pi = 138$ MeV, which is often neglected, and $m_K = 495$ MeV. The Wess-Zumino-Witten (WZW) term[8] is described by the action

$$\Gamma_{WZW} = -\frac{iN}{240\pi^2}\int_{\mathsf{M}} \mathrm{d}^5 r \epsilon^{\mu\nu\alpha\beta\gamma}\mathrm{tr}(l_\mu l_\nu l_\alpha l_\beta l_\gamma)$$

where $N$ is the number of colors, $l_\mu = U^\dagger\partial_\mu U$ and the integral is done on the five-dimensional manifold $\mathsf{M} = V \times S^1 \times I$ with the three-space volume $V$, the compactified time $S^1$ and the unit interval $I$ needed for a local form of WZW term.

Assuming maximal symmetry in the Skyrmion, we describe the hedgehog solution $U_0$ embedded in the SU(2) isospin subgroup of SU(3)

$$U_0(\vec{x}) = \left( \begin{array}{cc} e^{i\vec{\tau}\cdot\hat{x}f(r)} & 0 \\ 0 & 1 \end{array} \right)$$

where the $\tau_i$ ($i$=1,2,3) are Pauli matrices, $\hat{x} = \vec{x}/r$ and $f(r)$ is the chiral angle determined by minimizing the static mass $E$ given below and for unit winding number $\lim_{r\to\infty} f(r) = 0$ and $f(0) = \pi$. On the other hand, since the hedgehog ansatz has maximal or spherical symmetry, it is easily seen that spin plus isospin equals zero, so that isospin transformations and spatial rotations are related to each other.

Now we consider only the rigid motions of the SU(3) Skyrmion

$$U(\vec{x}, t) = \mathcal{A}(t) U_0(\vec{x}) \mathcal{A}(t)^\dagger$$

where, to separate the SU(2) rotations from the deviations into strange directions, the time-dependent rotations can be written as[9]

$$\mathcal{A}(t) = \left( \begin{array}{cc} A(t) & 0 \\ 0 & 1 \end{array} \right) S(t)$$

with $A(t) \in$ SU(2) and the small rigid oscillations $S(t)$ around the SU(2) rotations[9]. Furthermore, in the SU(2) subgroup of SU(3), the spin and isospin states can be treated by the time-dependent collective coordinates $a^\mu = (a^0, \vec{a})$ ($\mu = 0, 1, 2, 3$) corresponding to the spin and isospin rotations as in the standard SU(2) Skyrmion

$$A(t) = a^0 + i\vec{a} \cdot \vec{\tau}.$$

With the hedgehog ansatz and the collective rotation $A(t) \in$ SU(2) in the SU(2) embedding in the SU(3) manifold, the chiral field can be given by $U(\vec{x}, t) = A(t) U_0(\vec{x}) A^\dagger(t) = e^{i\tau_a R_{ab}\hat{x}_b f(r)}$ where $R_{ab} = \frac{1}{2}\mathrm{tr}(\tau_a A \tau_b A^\dagger)$.

On the other hand the small rigid oscillations $S$, which were also used in Ref. [4], can be described as

$$S(t) = \exp(i \sum_{a=4}^{7} d^a \lambda_a) = \exp(i\mathcal{D}),$$

where

$$\mathcal{D} = \begin{pmatrix} 0 & \sqrt{2}D \\ \sqrt{2}D^\dagger & 0 \end{pmatrix}, \quad D = \frac{1}{\sqrt{2}} \begin{pmatrix} d^4 - id^5 \\ d^6 - id^7 \end{pmatrix}.$$

After some algebra, the Skyrmion Lagrangian to order $1/N$ is then given in terms of the SU(2) collective coordinates $a^\mu$ and the strange deviations $D$

$$\begin{aligned} L \;=\; & -E - \frac{1}{2}\chi m_\pi^2 + 2\mathcal{I}_1 \dot{a}^\mu \dot{a}^\mu + 4\mathcal{I}_2 \dot{D}^\dagger \dot{D} + \frac{i}{2} N (D^\dagger \dot{D} - \dot{D}^\dagger D) \\ & -\chi(m_K^2 - m_\pi^2) D^\dagger D + 2i(\mathcal{I}_1 - 2\mathcal{I}_2)\{D^\dagger (a^0 \dot{\vec{a}} - \dot{a}^0 \vec{a} + \vec{a} \times \dot{\vec{a}}) \cdot \vec{\tau} \dot{D} \\ & -\dot{D}^\dagger (a^0 \dot{\vec{a}} - \dot{a}^0 \vec{a} + \vec{a} \times \dot{\vec{a}}) \cdot \vec{\tau} D\} - N D^\dagger (a^0 \dot{\vec{a}} - \dot{a}^0 \vec{a} + \vec{a} \times \dot{\vec{a}}) \cdot \vec{\tau} D \\ & +2(\mathcal{I}_1 - \frac{4}{3}\mathcal{I}_2)(D^\dagger D)(\dot{D}^\dagger \dot{D}) - \frac{1}{2}(\mathcal{I}_1 - \frac{4}{3}\mathcal{I}_2)(D^\dagger \dot{D} + \dot{D}^\dagger D)^2 \\ & +2\mathcal{I}_2 (D^\dagger \dot{D} - \dot{D}^\dagger D)^2 - \frac{i}{3} N (D^\dagger \dot{D} - \dot{D}^\dagger D) D^\dagger D \\ & +\frac{2}{3}\chi(m_K^2 - m_\pi^2)(D^\dagger D)^2 \end{aligned} \tag{2}$$

where the soliton energy $E$, the moments of inertia $\mathcal{I}_1$ and $\mathcal{I}_2$, and the strength $\chi$ of the chiral symmetry breaking are respectively given by

$$\begin{aligned} E &= 4\pi \int_0^\infty \mathrm{d}r r^2 [\frac{f_\pi^2}{2}((\frac{\mathrm{d}f}{\mathrm{d}r})^2 + \frac{2\sin^2 f}{r^2}) + \frac{1}{2e^2}\frac{\sin^2 f}{r^2}(2(\frac{\mathrm{d}f}{\mathrm{d}r})^2 + \frac{\sin^2 f}{r^2})], \\ \mathcal{I}_1 &= \frac{8\pi}{3} \int_0^\infty \mathrm{d}r r^2 \sin^2 f [f_\pi^2 + \frac{1}{e^2}((\frac{\mathrm{d}f}{\mathrm{d}r})^2 + \frac{\sin^2 f}{r^2})] \\ \mathcal{I}_2 &= 2\pi \int_0^\infty \mathrm{d}r r^2 (1 - \cos f)[f_\pi^2 + \frac{1}{4e^2}((\frac{\mathrm{d}f}{\mathrm{d}r})^2 + \frac{2\sin^2 f}{r^2})] \\ \chi &= 8\pi f_\pi^2 \int_0^\infty \mathrm{d}r r^2 (1 - \cos f). \end{aligned} \tag{3}$$

The momenta $\pi^\mu$ and $\pi_s^\alpha$, conjugate to the collective coordinates $a^\mu$ and the strange deviation $D_\alpha^\dagger$ are given by

$$\begin{aligned} \pi^0 &= 4\mathcal{I}_1 \dot{a}^0 - 2i(\mathcal{I}_1 - 2\mathcal{I}_2)(D^\dagger \vec{a} \cdot \vec{\tau} \dot{D} - \dot{D}^\dagger \vec{a} \cdot \vec{\tau} D) + N D^\dagger \vec{a} \cdot \vec{\tau} D \\ \vec{\pi} &= 4\mathcal{I}_1 \dot{\vec{a}} + 2i(\mathcal{I}_1 - 2\mathcal{I}_2)\{D^\dagger (a^0 \vec{\tau} - \vec{a} \times \vec{\tau}) \dot{D} - \dot{D}^\dagger (a^0 \vec{\tau} - \vec{a} \times \vec{\tau}) D\} \\ & \quad -N D^\dagger (a^0 \vec{\tau} - \vec{a} \times \vec{\tau}) D \\ \pi_s &= 4\mathcal{I}_2 \dot{D} - \frac{i}{2} N D - 2i(\mathcal{I}_1 - 2\mathcal{I}_2)(a^0 \dot{\vec{a}} - \dot{a}^0 \vec{a} + \vec{a} \times \dot{\vec{a}}) \cdot \vec{\tau} D \end{aligned}$$

$$
\begin{aligned}
&+2(\mathcal{I}_1 - \frac{4}{3}\mathcal{I}_2)(D^\dagger D)\dot{D} - (\mathcal{I}_1 - \frac{4}{3}\mathcal{I}_2)(D^\dagger \dot{D} + \dot{D}^\dagger D)D \\
&-4\mathcal{I}_2(D^\dagger \dot{D} - \dot{D}^\dagger D)D + \frac{i}{3}N(D^\dagger D)D
\end{aligned}
$$

which satisfy the Poisson brackets

$$
\{a^\mu, \pi^\nu\} = \delta^{\mu\nu}, \quad \{D^\dagger_\alpha, \pi^\beta_s\} = \{D^\beta, \pi^\dagger_{s,\alpha}\} = \delta^\beta_\alpha.
$$

Performing Legendre transformation, we obtain the Hamiltonian to order $1/N$ as follows

$$
\begin{aligned}
H &= E + \frac{1}{2}\chi m_\pi^2 + \frac{1}{8\mathcal{I}_1}\pi^\mu\pi^\mu + \frac{1}{4\mathcal{I}_2}\pi_s^\dagger\pi_s - i\frac{N}{8\mathcal{I}_2}(D^\dagger\pi_s - \pi_s^\dagger D) + (\frac{N^2}{16\mathcal{I}_2} \\
&+\chi(m_K^2 - m_\pi^2))D^\dagger D + i(\frac{1}{4\mathcal{I}_1} - \frac{1}{8\mathcal{I}_2})\{D^\dagger(a^0\vec{\pi} - \vec{a}\pi^0 + \vec{a}\times\vec{\pi})\cdot\vec{\tau}\pi_s \\
&-\pi_s^\dagger(a^0\vec{\pi} - \vec{a}\pi^0 + \vec{a}\times\vec{\pi})\cdot\vec{\tau}D\} + \frac{N}{8\mathcal{I}_2}D^\dagger(a^0\vec{\pi} - \vec{a}\pi^0 + \vec{a}\times\vec{\pi})\cdot\vec{\tau}D \\
&+(\frac{1}{2\mathcal{I}_1} - \frac{1}{3\mathcal{I}_2})(D^\dagger D)(\pi_s^\dagger\pi_s) + (\frac{1}{12\mathcal{I}_2} - \frac{1}{8\mathcal{I}_1})(D^\dagger\pi_s + \pi_s^\dagger D)^2 \\
&-\frac{1}{8\mathcal{I}_2}(D^\dagger\pi_s - \pi_s^\dagger D)^2 - i\frac{N}{8\mathcal{I}_2}(D^\dagger\pi_s - \pi_s^\dagger D)(D^\dagger D) \\
&+(\frac{N^2}{12\mathcal{I}_2} - \frac{2}{3}\chi(m_K^2 - m_\pi^2))(D^\dagger D)^2. \qquad (4)
\end{aligned}
$$

On the other hand, since the physical fields $a^\mu$ are the collective coordinates of the SU(2) group manifold isomorphic to the hypersphere $S^3$, we have the second class constraints

$$
\begin{aligned}
\Omega_1 &= a^\mu a^\mu - 1 \approx 0, \\
\Omega_2 &= a^\mu \pi^\mu \approx 0. \qquad (5)
\end{aligned}
$$

Here one notes that the derivation of the second constraint is not trivial differently from that in the SU(2) Skyrmion model[7] where the constraints (5) also hold. In other words, through the following complicated algebraic relations

$$
\{a^0, H\} = \frac{1}{4\mathcal{I}_1}\pi^0 - i(\frac{1}{4\mathcal{I}_1} - \frac{1}{8\mathcal{I}_2})(D^\dagger\vec{a}\cdot\vec{\tau}\pi_s - \pi_s^\dagger\vec{a}\cdot\vec{\tau}D) - \frac{N}{8\mathcal{I}_2}D^\dagger\vec{a}\cdot\vec{\tau}D
$$

$$
\begin{aligned}
\{\vec{a}, H\} &= \frac{1}{4\mathcal{I}_1}\vec{\pi} + i(\frac{1}{4\mathcal{I}_1} - \frac{1}{8\mathcal{I}_2})\{D^\dagger(a^0\vec{\tau} - \vec{a}\times\vec{\tau})\pi_s \\
&\quad -\pi_s^\dagger(a^0\vec{\tau} - \vec{a}\times\vec{\tau})D\} + \frac{N}{8\mathcal{I}_2}D^\dagger(a^0\vec{\tau} - \vec{a}\times\vec{\tau})D,
\end{aligned}
$$

we can obtain the Poisson commutator

$$\{\Omega_1, H\} = \frac{1}{2\mathcal{I}}\Omega_2$$

which yields the second constraint of (5) since the secondary constraint comes from the time evolution of $\Omega_1$. The above two constraints then yield the Poisson algebra

$$\Delta_{kk'} = \{\Omega_k, \Omega_{k'}\} = 2\epsilon^{kk'}a^\mu a^\mu$$

with $\epsilon^{12} = -\epsilon^{21} = 1$. Using the Dirac bracket [10] defined by

$$\{A, B\}_D = \{A, B\} - \{A, \Omega_k\}\Delta^{kk'}\{\Omega_{k'}, B\}$$

with $\Delta^{kk'}$ being the inverse of $\Delta_{kk'}$ one can obtain the commutator relations

$$
\begin{aligned}
\{a^\mu, a^\nu\}_D &= 0, \\
\{a^\mu, \pi^\nu\}_D &= \delta^{\mu\nu} - \frac{a^\mu a^\nu}{a^\sigma a^\sigma}, \\
\{\pi^\mu, \pi^\nu\}_D &= \frac{1}{a^\sigma a^\sigma}(a^\nu\pi^\mu - a^\mu\pi^\nu).
\end{aligned}
$$

Now, maintaining the SU(2) symmetry originated from the massless $u$- and $d$-quarks and following the abelian BFT formalism [6, 7] which systematically converts the second class constraints into first class ones, we introduce two auxiliary fields $\Phi^i$ corresponding to $\Omega_i$ with the Poisson brackets

$$\{\Phi^i, \Phi^j\} = \epsilon^{ij}.$$

The first class constraints $\tilde{\Omega}_i$ are then constructed as a power series of the auxiliary fields:

$$\tilde{\Omega}_i = \sum_{n=0}^{\infty}\Omega_i^{(n)}, \qquad \Omega_i^{(0)} = \Omega_i \tag{6}$$

where $\Omega_i^{(n)}$ are polynomials in the auxiliary fields $\Phi^j$ of degree $n$, to be determined by the requirement that the first class constraints $\tilde{\Omega}_i$ satisfy an abelian algebra as follows

$$\{\tilde{\Omega}_i, \tilde{\Omega}_j\} = 0. \tag{7}$$

After some algebraic manipulations, one can obtain the desired first class constraints

$$\begin{aligned}\tilde{\Omega}_1 &= \Omega_1 + 2\Phi^1, \\ \tilde{\Omega}_2 &= \Omega_2 - a^\mu a^\mu \Phi^2\end{aligned}$$

which yield the strongly involutive first class constraint algebra (7). On the other hand, the corresponding first class Hamiltonian is given by

$$\begin{aligned}\tilde{H} &= E + \frac{1}{2}\chi m_\pi^2 + \frac{1}{8\mathcal{I}_1}(\pi^\mu - a^\mu\Phi^2)(\pi^\mu - a^\mu\Phi^2)\frac{a^\nu a^\nu}{a^\nu a^\nu + 2\Phi^1} \\ &+\frac{1}{4\mathcal{I}_2}\pi_s^\dagger\pi_s - i\frac{N}{8\mathcal{I}_2}(D^\dagger\pi_s - \pi_s^\dagger D) + (\frac{N^2}{16\mathcal{I}_2} + \chi(m_K^2 - m_\pi^2))D^\dagger D \\ &+i(\frac{1}{4\mathcal{I}_1} - \frac{1}{8\mathcal{I}_2})\{D^\dagger(a^0\vec{\pi} - \vec{a}\pi^0 + \vec{a}\times\vec{\pi})\cdot\vec{\tau}\pi_s \\ &-\pi_s^\dagger(a^0\vec{\pi} - \vec{a}\pi^0 + \vec{a}\times\vec{\pi})\cdot\vec{\tau}D\} + \frac{N}{8\mathcal{I}_2}D^\dagger(a^0\vec{\pi} - \vec{a}\pi^0 + \vec{a}\times\vec{\pi})\cdot\vec{\tau}D \\ &+\cdots \end{aligned} \tag{8}$$

where the ellipsis stands for the strange-strange interaction terms of order $1/N$ which can be readily read off from Eq. (4). Here one notes that the BFT corrections for the second class constraints do not affect even the isospin-strange coupling terms with the Pauli matrices $\tau_i$. The above first class Hamiltonian is also strongly involutive with the first class constraints

$$\{\tilde{\Omega}_i, \tilde{H}\} = 0.$$

Now the substitution of the first class constraints into the Hamiltonian (8) yields the Hamiltonian of the form

$$\begin{aligned}\tilde{H} &= E + \frac{1}{2}\chi m_\pi^2 + \frac{1}{8\mathcal{I}_1}(a^\mu a^\mu \pi^\nu\pi^\nu - a^\mu\pi^\mu a^\nu\pi^\nu) \\ &+\frac{1}{4\mathcal{I}_2}\pi_s^\dagger\pi_s - i\frac{N}{8\mathcal{I}_2}(D^\dagger\pi_s - \pi_s^\dagger D) + (\frac{N^2}{16\mathcal{I}_2} + \chi(m_K^2 - m_\pi^2))D^\dagger D \\ &+i(\frac{1}{4\mathcal{I}_1} - \frac{1}{8\mathcal{I}_2})\{D^\dagger(a^0\vec{\pi} - \vec{a}\pi^0 + \vec{a}\times\vec{\pi})\cdot\vec{\tau}\pi_s \\ &-\pi_s^\dagger(a^0\vec{\pi} - \vec{a}\pi^0 + \vec{a}\times\vec{\pi})\cdot\vec{\tau}D\} + \frac{N}{8\mathcal{I}_2}D^\dagger(a^0\vec{\pi} - \vec{a}\pi^0 + \vec{a}\times\vec{\pi})\cdot\vec{\tau}D \\ &+\cdots. \end{aligned} \tag{9}$$

Following the symmetrization procedure[7], we obtain the Weyl ordering correction to the first class Hamiltonian (9) as follows

$$
\begin{aligned}
\tilde{H} \;=\; & E + \frac{1}{2}\chi m_\pi^2 + \frac{1}{2\mathcal{I}_1}(\vec{I}^2 + \frac{1}{4}) + \frac{1}{4\mathcal{I}_2}\pi_s^\dagger \pi_s - i\frac{N}{8\mathcal{I}_2}(D^\dagger \pi_s - \pi_s^\dagger D) \\
& + (\frac{N^2}{16\mathcal{I}_2} + \chi(m_K^2 - m_\pi^2))D^\dagger D + i(\frac{1}{2\mathcal{I}_1} - \frac{1}{4\mathcal{I}_2})(D^\dagger \vec{I}\cdot\vec{\tau}\pi_s - \pi_s^\dagger \vec{I}\cdot\vec{\tau} D) \\
& + \frac{N}{4\mathcal{I}_2} D^\dagger \vec{I}\cdot\vec{\tau} D + \cdots . \qquad (10)
\end{aligned}
$$

where, as in the SU(2) standard Skyrmion, the isospin operator $\vec{I}$ is given by[1]

$$\vec{I} = \frac{1}{2}(a^0\vec{\pi} - \vec{a}\pi^0 + \vec{a}\times\vec{\pi})$$

which itself is invariant under the Weyl ordering procedure. Here, by using the SU(2) collective coordinates $a^\mu$ instead of the angular velocity of the SU(2) rotation $A^\dagger \dot{A} = \frac{i}{2}\dot{\vec{\alpha}}\cdot\vec{\tau}$ [1] used in Ref.[4], we have obtained the same result (10) as that of Klebanov and Westerberg (KW) [4], apart from the overall energy shift $\frac{1}{8\mathcal{I}_1}$ originated from the BFT correction.

Following the quantization scheme of KW for the strangeness flavor direction, one can obtain the Hamiltonian of the form

$$
\begin{aligned}
\tilde{H} \;=\; & E + \frac{1}{2}\chi m_\pi^2 + \frac{1}{2\mathcal{I}_1}(\vec{I}^2 + \frac{1}{4}) + \frac{N}{8\mathcal{I}_2}(\mu - 1)a^\dagger a \\
& + (\frac{1}{2\mathcal{I}_1} - \frac{1}{4\mathcal{I}_2\mu}(\mu - 1))a^\dagger \vec{I}\cdot\vec{\tau} a + (\frac{1}{8\mathcal{I}_1} - \frac{1}{8\mathcal{I}_2\mu^2}(\mu - 1))(a^\dagger a)^2 \qquad (11)
\end{aligned}
$$

where

$$
\begin{aligned}
\mu &= (1 + \frac{m_K^2 - m_\pi^2}{m_0^2})^{1/2} \\
m_0 &= \frac{N}{4(\chi\mathcal{I}_2)^{1/2}}
\end{aligned}
$$

and $a^\dagger$ is creation operator for constituent strange quarks and we have ignored the irrelevant creation operator $b^\dagger$ for strange antiquarks[4]. Introducing the

[1] Here one notes that the collective coordinates $a^\mu$ are associated with the angular velocity of the SU(2) rotation $\dot{\vec{\alpha}}$ through the conversion relation $\dot{a}^\mu \dot{a}^\mu = \frac{1}{4}\dot{\vec{\alpha}}\cdot\dot{\vec{\alpha}}$.

angular momentum of the strange quarks

$$\vec{J}_s = \frac{1}{2}a^\dagger\vec{\tau}a,$$

one can rewrite the Hamiltonian (11) as

$$\tilde{H} = E + \frac{1}{2}m_\pi^2 + \omega a^\dagger a + \frac{1}{2\mathcal{I}_1}(\vec{I}^2 + 2c\vec{I}\cdot\vec{J}_s + \bar{c}\vec{J}_s^2 + \frac{1}{4}) \tag{12}$$

where

$$\begin{aligned}
\omega &= \frac{N}{8\mathcal{I}_2}(\mu - 1) \\
c &= 1 - \frac{\mathcal{I}_1}{2\mathcal{I}_2\mu}(\mu - 1) \\
\bar{c} &= 1 - \frac{\mathcal{I}_1}{\mathcal{I}_2\mu^2}(\mu - 1).
\end{aligned}$$

The Hamiltonian (12) then yields the structure of the hyperfine splittings as follows

$$\begin{aligned}
\delta M &= \frac{1}{2\mathcal{I}_1}[cJ(J+1) + (1-c)(I(I+1) - \frac{Y^2-1}{4}) \\
&\quad + (1 + \bar{c} - 2c)\frac{Y^2-1}{4} + \frac{1}{4}(1 + \bar{c} - c)]
\end{aligned}$$

where $\vec{J} = \vec{I} + \vec{J}_s$ is the total angular momentum of the quarks.

Next, using the Weyl ordering corrected (WOC) energy spectrum (12), we easily obtain the hyperfine structure of the nucleon and delta hyperon masses to yield the soliton energy and the moment of inertia

$$\begin{aligned}
E &= \frac{1}{3}(4M_N - M_\Delta) \\
\mathcal{I} &= \frac{3}{2}(M_\Delta - M_N)^{-1}.
\end{aligned} \tag{13}$$

Substituting the experimental values $M_N = 939$ MeV and $N_\Delta = 1232$ MeV into Eq. (13) and using the expressions for $E$ and $\mathcal{I}_1$ given in Eq. (3), one can predict the pion decay constant $f_\pi$ and the Skyrmion parameter $e$ in the Weyl ordering corrected rigid rotator approach as follows

$$f_\pi = 52.9 \text{ MeV}, \quad e = 4.88.$$

With these fixed values of $f_\pi$ and $e$, one can then proceed to evaluate the inertia parameters as follows

$$\mathcal{I}_1^{-1} = 198 \text{ MeV}, \ \mathcal{I}_2^{-1} = 613 \text{ MeV}, \ \chi^{-1} = 257 \text{ MeV}, \ E = 840 \text{ MeV}$$

to yield the predictions for the values of $c$ and $\bar{c}$

$$c = 0.27, \quad \bar{c} = 0.23 \tag{14}$$

which are contained in Table 1, together with the experimental data and the standard SU(3) rigid rotator and bound state approach predictions.[2] Here one notes that the massless SU(3) Skyrmions have the same values of $c$ and $\bar{c}$ both in the standard and WOC cases since the chiral angles are the same in these cases. However, in the massive Skyrmions the equation of motion for the chiral angle has an additional term proportional to $(m_\pi/ef_\pi)^2$[11] to yield the discrepancies between the two chiral angles of the standard and WOC cases since the standard Skyrmion has the values $f_\pi = 54.0$ MeV and $e = 4.84$ different from the above ones in the massive WOC Skyrmion. With these chiral angles and values of $f_\pi$ and $e$, one can obtain different sets of $c$ and $\bar{c}$ in the massive standard and WOC Skyrmions, which are about 5% improved with respect to those of the massless Skyrmions as shown in Table 1.

Now we investigate the relations between the Hamiltonian (12) and the Berry phases[12]. In the Berry phase approach to the SU(3) Skyrmion, the Hamiltonian takes the simple form[13]

$$H^* = \epsilon_K + \frac{1}{8\mathcal{I}_1}(\vec{R}^2 - 2g_K \vec{R}\cdot\vec{T}_K + g_K^2 \vec{T}_K^2) \tag{15}$$

where $\epsilon_K$ is the eigenenergy in the $K$ state, $g_K$ is the Berry charge, $\vec{R}$ ($\vec{L}$) is the right (left) generators of the group $SO(4) \approx SU(2) \times SU(2)$ and $\vec{T}_K$ is the angular momentum of the "slow" rotation. We recall that $\vec{I} = \frac{\vec{L}}{2} = -\frac{\vec{R}}{2}$

[2] Here we have the modified predictions $c = 0.22$ and $\bar{c} = 0.34$ of the standard rigid rotator without pion mass since the numerical evaluation for the inertia parameters should be fixed with the values $\mathcal{I}_1^{-1} = 196$ MeV, $\mathcal{I}_2^{-1} = 528$ MeV, $\chi^{-1} = 182$ MeV and $E = 866$ MeV, instead of $\mathcal{I}_1^{-1} = 211$ MeV, $\mathcal{I}_2^{-1} = 552$ MeV, $\chi^{-1} = 202$ MeV and $E = 862$ MeV which yields $c = 0.28$ and $\bar{c} = 0.35$[4], to be consistent with the parameter fit $f_\pi = 64.5$ MeV, $e = 5.45$ used in the massless standard SU(2) Skyrmion[1]. Also one notes that the bound state approach does not include the quartic terms in the kaon field.

and $\vec{L}^2 = \vec{R}^2$ on $S^3$. Applying the BFT scheme to the Hamiltonian (15) we can obtain the Hamiltonian of the form

$$\tilde{H}^* = \epsilon_K + \frac{1}{2\mathcal{I}_1}(\vec{I}^2 + g_K \vec{I}\cdot\vec{T}_K + (\frac{g_K}{2})^2 \vec{T}_K^2 + \frac{1}{4}). \tag{16}$$

In the case with the relation $\bar{c} = c^2$, the Hamiltonian (12) is equivalent to $\tilde{H}^*$ in the Berry phase approach where the corresponding physical quantities can be read off as follows

$$\begin{aligned} \epsilon_K &= E + \frac{1}{2}\chi m_\pi^2 + \omega a^\dagger a \\ \vec{T}_K &= \vec{J}_s \\ g_K &= 2c. \end{aligned} \tag{17}$$

The same case with the Hamiltonian (16) follows from the quark model and the bound state approach with the quartic terms in the kaon field neglected. In fact, the strange-strange interactions in the Hamiltonian (12) break these relations to yield the numerical values of $\bar{c}$ in Table 1.

Next, in order to take into account the missing order $N^0$ effects, we consider the Casimir energy contributions to the Hamiltonian (12). The Casimir energy originated from the meson fluctuation can be given by the phase shift formula[14, 15]

$$\begin{aligned} E_C(\mu) &= \frac{1}{2\pi}\sum_{i=\pi,K}\{\int_0^\infty \mathrm{d}p[-\frac{p}{\sqrt{p^2+m_i^2}}(\delta(p) - \bar{a}_0 p^3 - \bar{a}_1 p) + \frac{\bar{a}_2}{\sqrt{p^2+\mu^2}}] \\ &\quad -\frac{3}{8}\bar{a}_0 m_i^4(\frac{3}{4} + \frac{1}{2}\ln\frac{\mu^2}{m_i^2}) + \frac{1}{4}\bar{a}_1 m_i^2(1 + \ln\frac{\mu^2}{m_i^2}) - m_i\delta(0)\} \\ &\quad + \cdots \end{aligned} \tag{18}$$

where the ellipsis denotes the contributions from the counter terms and the bound states (if any). Here $\mu$ is the energy scale and $\delta(p)$ is the phase shift with the momentum $p$ and the coefficients $\bar{a}_i$ $(i = 0, 1, 2)$ are defined by the asymptotic expansion of $\delta'(p)$, namely, $\delta'(p) = 3\bar{a}_0 p^2 + \bar{a}_1 - \frac{\bar{a}_2}{p^2} + \cdots$. Even though the Casimir energy correction does not contribute to the ratios $c$ and $\bar{c}$ associated with the order $1/N$ piece of the Hamiltonian (12), these effects seem to be significant in other physical quantities such as the H dibaryon mass[5] which will be studied elsewhere.

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Table 1: The values of $c$ and $\bar{c}$ in the bound state and the standard and Weyl ordering corrected (WOC) rigid rotator approaches to the massless and massive SU(3) Skyrmions compared with experimental data.

| Source | $c$ | $\bar{c}$ |
|---|---|---|
| Bound state, partial | 0.60 | 0.36 |
| Rigid rotator, massless standard | 0.22 | 0.34 |
| Rigid rotator, massless WOC | 0.22 | 0.34 |
| Rigid rotator, massive standard | 0.26 | 0.23 |
| Rigid rotator, massive WOC | 0.27 | 0.23 |
| Experiment | 0.67 | 0.27 |